A Level Further Mathematics A
Y543 Mechanics
Sample Question Paper   Version 2

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

You must have:
• Printed Answer Booklet
• Formulae A Level Further Mathematics A

You may use:
• a scientific or graphical calculator

INSTRUCTIONS

• Use black ink. HB pencil may be used for graphs and diagrams only.
• Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
• Answer **all** the questions.
• **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
• Do **not** write in the bar codes.
• You are permitted to use a scientific or graphical calculator in this paper.
• Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
• The acceleration due to gravity is denoted by $g \, \text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

• The total number of marks for this paper is **75**.
• The marks for each question are shown in brackets **[ ]**.
• **You are reminded of the need for clear presentation in your answers.**
• The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1   A body, *P*, of mass 2 kg moves under the action of a single force **F** N. At time *t* s, the velocity of the body is
    v m s^{-1}, where

$$\mathbf{v} = \left(t^2 - 3\right)\mathbf{i} + \frac{5}{2t+1}\mathbf{j} \text{ for } t \geq 2.$$

   (i) Obtain **F** in terms of *t*. **[3]**

   (ii) Calculate the rate at which the force **F** is working at $t = 4$. **[3]**

   (iii) By considering the change in kinetic energy of *P*, calculate the work done by the force **F** during the
    time interval $2 \leq t \leq 4$. **[3]**

2   As part of a training exercise an army recruit of mass 75 kg falls a vertical distance of 5 m before landing
    on a mat of thickness 1.2 m. The army recruit sinks a distance of *x* m into the mat before instantaneously
    coming to rest. The mat can be modelled as a spring of natural length 1.2 m and modulus of elasticity
    10 800 N and the army recruit can be modelled as a particle falling vertically with an initial speed
    of 2 m s^{-1}.

   (i) Show that *x* satisfies the equation $300x^2 - 49x - 255 = 0$. **[5]**

   (ii) Calculate the value of *x*. **[2]**

   (iii) Ignoring the possible effect of air resistance, make
- one comment on the assumptions made and,
- suggest a possible refinement to the model. **[2]**

3   A body, *Q*, of mass 2 kg moves in a straight line under the action of a single force which acts in the
    direction of motion of *Q*. Initially the speed of *Q* is 5 m s^{-1}. At time *t* s, the magnitude *F* N of the force is
    given by

$$F = t^2 + 3e^t, \quad 0 \leq t \leq 4.$$

   (i) Calculate the impulse of the force over the time interval. **[3]**

   (ii) Hence find the speed of *Q* when $t = 4$. **[2]**

4 A light inextensible taut rope, of length $4\,\mathrm{m}$, is attached at one end A to the centre of the horizontal ceiling of a gym. The other end of the rope B is being held by a child of mass $35\,\mathrm{kg}$. Initially the child is held at rest with the rope making an angle of $60°$ to the downward vertical and it may be assumed that the child can be modelled as a particle attached to the end of the rope. The child is released at a height $5\,\mathrm{m}$ above the horizontal ground.

   (i) Show that the speed, $v\,\mathrm{m\,s^{-1}}$, of the child when the rope makes an angle θ with the downward vertical is given by $v^2 = 4g(2\cos\theta - 1)$. **[5]**

   (ii) At the instant when $\theta = 0°$, the child lets go of the rope and moves freely under the influence of gravity only. Determine the speed and direction of the child at the moment that the child reaches the ground. **[5]**

   (iii) The child returns to the initial position and is released again from rest. Find the value of θ when the tension in the rope is three times greater than the tension in the rope at the instant the child is released. **[5]**

5 A particle P of mass $m\,\mathrm{kg}$ is projected vertically upwards through a liquid. Student A measures P's initial speed as $(8.5 \pm 0.25)\,\mathrm{m\,s^{-1}}$ and they also record the time for P to attain its greatest height above the initial point of projection as over 3 seconds.

   In an attempt to model the motion of P student B determines that t seconds after projection the only forces acting on P are its weight and the resistance from the liquid. Student B models the resistance from the liquid to be of magnitude $mv^2 - 6mv$, where v is the speed of the particle.

   (i) (a) Show that $\dfrac{\mathrm{d}t}{\mathrm{d}v} = -\dfrac{1}{(v-3)^2 + 0.8}$. **[3]**

   (b) Determine whether student B's model is consistent with the time recorded by A for P to attain its greatest height. **[5]**

   After attaining its greatest height P now falls through the liquid. Student C claims that the time taken for P to achieve a speed of $1\,\mathrm{m\,s^{-1}}$ when falling through the liquid is given by

   $$-\int_0^1 \frac{1}{(v-3)^2 + 0.8}\,\mathrm{d}v.$$

   (ii) Explain why student C's claim is incorrect and write down the integral which would give the correct time for P to achieve a speed of $1\,\mathrm{m\,s^{-1}}$ when falling through the liquid. **[3]**

6 Two uniform smooth spheres *A* and *B* of equal radius are moving on a smooth horizontal surface when they collide. *A* has mass 2.5 kg and *B* has mass 3 kg. Immediately before the collision *A* and *B* each has speed u m s^{-1} and each moves in a direction at an angle θ to their line of centres, as indicated in Fig. 1. Immediately after the collision *A* has speed v_1 m s^{-1} and moves in a direction at an angle α to the line of centres, and *B* has speed v_2 m s^{-1} and moves in a direction at an angle β to the line of centres as indicated in Fig. 2. The coefficient of restitution between *A* and *B* is *e*.
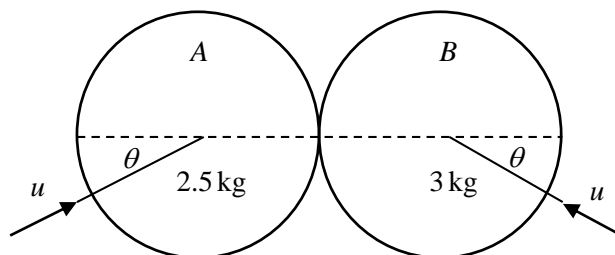
Fig. 1
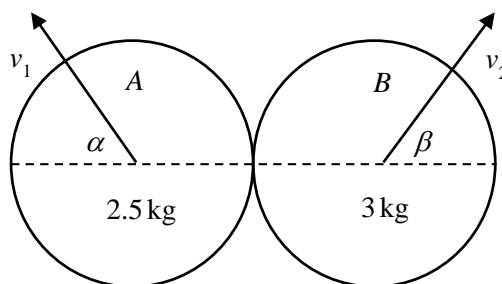
Fig. 2

(i) Show that $\tan\beta = \dfrac{11\tan\theta}{10e-1}$. **[8]**

(ii) Given that after impact sphere *A* moves at an angle of $50°$ to the line of centres and *B* moves perpendicular to the line of centres, find θ. **[4]**
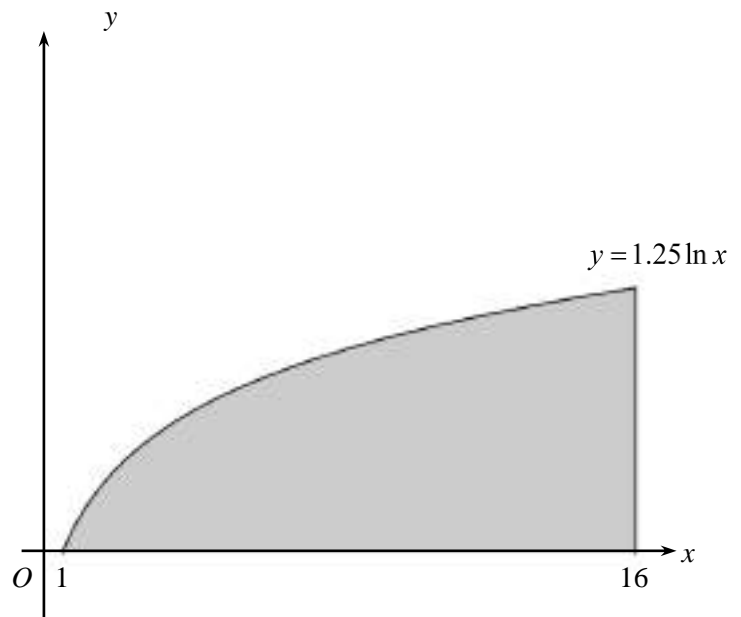
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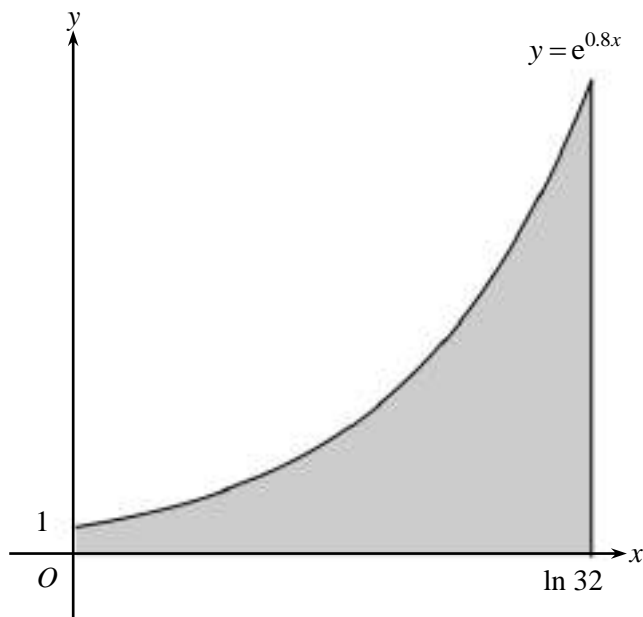
Fig. 3

Fig. 4

The region bounded by the x-axis, the y-axis, the line $x = \ln 32$ and the curve $y = e^{0.8x}$ for $0 \le x \le \ln 32$, is occupied by a uniform lamina (see Fig. 3).

(i) Show that the x-coordinate of the centre of mass of the lamina is given by $\frac{16}{3} \ln 2 - \frac{5}{4}$. **[7]**

(ii) Calculate the y-coordinate of the centre of mass of the lamina. **[3]**

(iii) The region bounded by the x-axis, the line $x = 16$ and the curve $y = 1.25 \ln x$ for $1 \le x \le 16$, is occupied by a second uniform lamina (see Fig. 4). By using your answer to part **(i)** find, to 3 significant figures, the x-coordinate of the centre of mass of this second lamina. **[4]**

END OF QUESTION PAPER

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